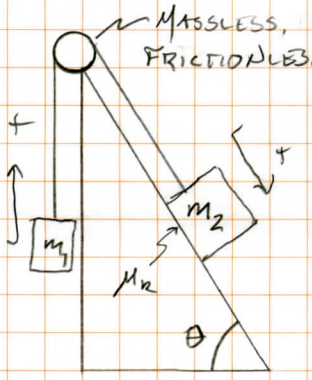
TM 5 PR. 2.32

TM5 2-32 FIND THE ANGLE THAT ALLOWS THE BLOCKS TO MOVE AT CONSTANT SPEED FOR FRICTION COEFFICIENT μ_k.

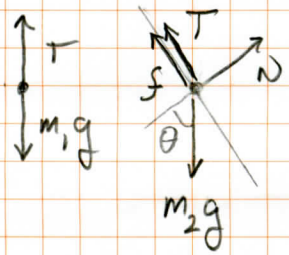


m_1: $\Sigma F_{VERT} = m_1 a_{VERT}^{\,0}$

$T - m_1 g = 0 \Rightarrow T = m_1 g = mg$

m_2: $\Sigma F_{\perp} = m_2 a_{\perp}^{\,0}$

$N - m_2 g\cos\theta = 0 \Rightarrow N = m_2 g\cos\theta = 2mg\cos\theta$



$\Sigma F_{\parallel} = m_2 a_{\parallel}^{\,0}$

$m_2 g\sin\theta - T - f = 0$

$2mg\sin\theta - mg - 2\mu_k mg\cos\theta = 0$

$2\mu_k\cos\theta = 2\sin\theta - 1$

$4\mu_k^2\cos^2\theta = 4\sin^2\theta - 4\sin\theta + 1$

$4\mu_k^2 - 4\mu_k^2\sin^2\theta = 4\sin^2\theta - 4\sin\theta + 1$

$(4 + 4\mu_k^2)\sin^2\theta - 4\sin\theta + (1 - 4\mu_k^2) = 0$

$$\sin\theta = \frac{4 \pm \sqrt{16 - 16(1+\mu_k^2)(1-4\mu_k^2)}}{8(1-\mu_k^2)} = \frac{1 \pm \sqrt{1 - (1+\mu_k^2)(1-4\mu_k^2)}}{2(1-\mu_k^2)}$$

$$\sin\theta = \frac{1 \pm \sqrt{1 - (1 - 4\mu_k^2 + \mu_k^2 - 4\mu_k^4)}}{2(1-\mu_k^2)} = \frac{1 \pm \sqrt{1 - 1 + 4\mu_k^2 - \mu_k^2 + 4\mu_k^4}}{2(1-\mu_k^2)}$$

$$\boxed{\sin\theta = \frac{1 \pm \mu_k\sqrt{3 + 4\mu_k^2}}{2(1-\mu_k^2)}} \quad \text{QED}$$